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## An electrostatic MEMS frequency up-converter for efficient energy harvesting

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### Abstract

The design of an MEMS frequency up-converter is described in this paper. The device consists of two electrostatically coupled resonators that relies on the hysteresis of electrostatic pull-in in order to multiply the input excitation frequency by a significant factor, possibly up to several orders of magnitudes in order to increase the efficiency of electrostatic energy harvesting. An analytical solution of the performance of the device is obtained using a spring-mass model, and the optimal values of physical parameters are derived analytically.

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### 1. Introduction

The highly appealing prospect of supplying ultra-low power electronics with the necessary energy by integrating an energy-scavenging device is most often impaired by the low efficiency of energy harvesters, this issue is further exacerbated by the lack of high frequency environmental excitation where mechanical-to-electrical energy conversion is most practical. In literature, several frequency up-conversion MEMS devices have been suggested that rely on either shocks or force nonlinearities [1]–[3].

In this paper an efficient frequency up-converter that exploits the hysteretic effect of electrostatic pull-in between two resonators is presented, in addition to providing efficient frequency up-conversion by transferring the energy between two resonators it is possible to individually optimize the performance of each resonator for either vibrational energy capture or mechanical-to-electrical energy transduction.

**Nomenclature**

$A$	surface area for electrostatic actuation
$a_0$	input acceleration to the seismic (primary resonator)
$C(X)$	position dependent capacitance between the two resonators
$\delta$	maximum vibration amplitude of the primary resonator
$E_T$	total energy transferred converted from low to high frequency resonator
$E_{elastic}$	elastic strain energy
$E_{elec}$	electrostatic potential well that the secondary resonator needs to overcome as it oscillates after detachment
$E_{pull-in}$	energy lost during the pull-in phase between the two resonators
$\epsilon_0$	dielectric constant of vacuum
$\epsilon_r$	relative dielectric constant
$F_{elec}$	electrostatic force between the two resonators
$h$	thickness of dielectric layer
$K$	spring constant of the secondary resonator
$K_S$	spring constant of the seismic (primary) resonator
$M$	effective mass of the secondary resonator
$M_S$	effective mass of the seismic (primary) resonator
$Q$	quality factor of the seismic (primary) resonator
$X$	separation between the two resonators
$X_{cr}$	critical displacement of the secondary resonator upon which the two resonators break contact
$V$	applied voltage
$\omega_0$	resonance frequency of seismic (primary) resonator

**2. Modeling of the MEMS frequency up-converter**

The MEMS device used in this work is depicted schematically in Fig. 1, where it consists of two MEMS resonators: the first is a low frequency resonator (seismic resonator) with a natural frequency within the bandwidth of the input excitation represented in Fig. 1 by the mass-spring system denoted  $M_S$  and  $K_S$  respectively. The second high frequency resonator, is depicted with the lumped elements  $K$  and  $M$  in Fig. 1.

The seismic mass is covered by a dielectric layer of thickness  $h$  and a relative dielectric constant  $\epsilon_r$ , and a constant voltage difference  $V$  is maintained between the two resonators whose respective proof mass constitute the electrodes of a parallel plate capacitor.

The attractive force between the two electrodes (formed by the masses  $M$  and  $M_S$ ) is thus given by:

$$F_{elec} = \frac{\epsilon_0 A V^2}{2 \left( \frac{h}{\epsilon_r} + X \right)^2} \quad (1)$$

Upon external excitation of the seismic resonator the seismic mass moves to a sufficiently near distance to the high frequency resonator to cause a pull-in effect [4]. The high frequency oscillator is thereafter pulled down as the seismic resonator returns to an equilibrium position until the electrostatic force is no longer sufficient to hold the two electrodes stuck at which point and due to the hysteretic effect of electrostatic pull-in, the high frequency oscillator is released and oscillates with large amplitudes near its natural frequency ("near" because it is situated in an electrostatic potential well which results in nonlinear oscillations), these oscillations are thereafter converted to electrical energy using a separate transduction system.

It is important at this point to identify the effect responsible for converting energy into high frequency domain not as the impact caused by pull-in, ideally that impact will carry very little energy, but by the entrainment and sudden release

of the proof mass of the high frequency resonator as caused by the hysteresis of the electrostatic force.

By noting the critical position at which the oscillator is released as  $X_{cr}$ , it is possible to express  $X_{cr}$  using the equilibrium of electrostatic and elastic force, i.e.  $F_{elec} = F_{elastic} = KX_{cr}$ .

By assuming that the seismic resonator is driven near its natural frequency  $\omega_0$ , and denoting its maximum displacement as  $\delta$ . It is possible to express the equation describing the common motion of the two resonators (once they are stuck) as:

$$\ddot{X} + \frac{K + K_S}{M + M_S} X = \frac{K_S}{M + M_S} \delta \quad (2)$$

The above equation has the following solution  $X(t) = \frac{Qa_0M_S}{K + K_S} (1 - \cos(\omega_I t))$ , where  $\omega_I = \sqrt{\frac{K + K_S}{M + M_S}}$ .

The energy transferred from the seismic resonator to the high frequency oscillator is expressed as follows:

$$E_T = E_{elastic} - E_{elect} - E_{pull-in} \quad (3)$$

Note that  $E_{elec}$  need not be completely lost, however, equation (3) represents an upper limit on dissipation due to the electrostatic potential well.

Upon explicitly calculating each of the dissipation terms, it is possible to obtain an expression for the energy up-converted in the frequency domain as follows:

$$E_T = KX_{cr} \left( \frac{X_{cr}}{2} - \frac{3h}{\epsilon_r} \right) \quad (4)$$

By injecting the expression for the time dependent deformation into (4) and deriving with respect to time, it is possible to obtain the detachment length that maximizes  $E_T$  as:

$$X_{cr} = \frac{2Qa_0M_S}{K + K_S} \quad (6)$$

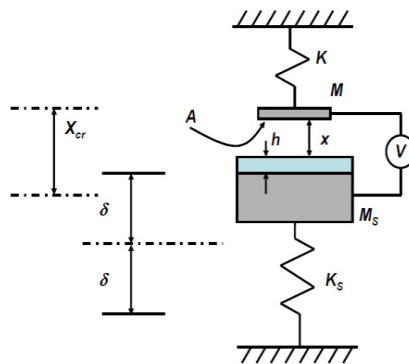


Fig. 1. Schematic representation of the MEMS frequency up-converter, a reduced order model identifying the main physical parameters and the magnitude of oscillation of the seismic mass ( $\delta$ ) and the detachment length ( $X_{cr}$ ).

As a case study, compliant with cardiac medical application, a system with the following properties is considered  $M_S$

$= 2g$ ,  $\omega_0 = 45$ ,  $K_S = 4 \text{ N/m}$ ,  $Q = 1$ ,  $a_0 = 0.1 \text{ m/s}^2$ ,  $A = 10^{-6} \text{ m}^2$ , and  $\epsilon_r = 3.9$  (equivalent to that of  $\text{SiO}_2$ ) resulting in  $X_{cr} = 50 \mu\text{m}$ , and  $V = 2V$  ( $h = 1 \mu\text{m}$ ).

The dynamics of the above system were implemented using a 1-dimensional finite time difference algorithm implemented in MATLAB. The time evolution of the system is shown in Fig. 2, where the graph shows the seismic resonator starting with an initial displacement amplitude  $\delta = g$ , where  $g$  is the initial gap between the two proof mass. The plot in Fig. 2 identifies the pull-in, the entrainment of the high-frequency proof mass along with the seismic resonator, and the detachment of the two electrodes (constituted by the proof mass) at the critical displacement. Once the two resonators are detached, most of the oscillation energy is transferred from the low-frequency seismic into the high-frequency resonator, as can be noted from the respective amplitudes of vibration after detachment.

The converted energy and the efficiency of the conversion,  $\eta$ , is plotted in Fig. 3 as a function of  $K/K_S$ . Note that if the term  $(h/\epsilon_r)$  is negligibly small compared to gap, the efficiency approaches 100% this is demonstrated in Fig. 4 when comparing the efficiencies of  $h = 1 \mu\text{m}$  and  $h = 10 \mu\text{m}$ .

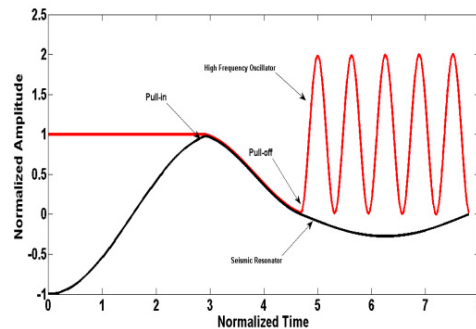


Fig. 2. Dynamics of the system where the seismic resonator (black) starts with an initial displacement, and the high frequency resonator (red) starts from rest, plotted as a function of normalized time.

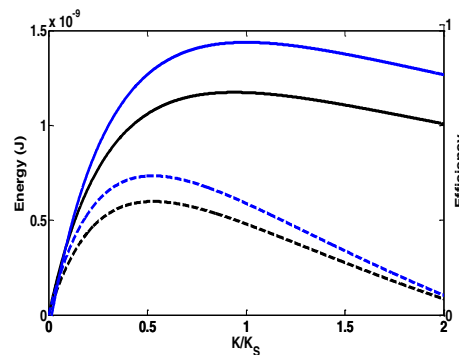


Fig. 3. Graphical representation showing the energy (black lines), and efficiency (blue lines) for  $h = 1 \mu\text{m}$  (solid line), and  $h = 10 \mu\text{m}$  (dashed lines).

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